Correspondence

Sufficient conditions for the asymptotic stability of interval matrices

M. MANSOUR†

Jiang (1987) gave a sufficient condition for the asymptotic stability of interval matrices. We shall show here that his results can be derived in a trivial way using the Lyapunov equation. An extension of his results and a similar result for discrete systems is stated.

1. Introduction

Consider first the Hurwitz stability of

\[ A = \sum_{i=1}^{k} \beta_i A_i \quad \beta_i \geq 0 \]

**Theorem 1**

A sufficient condition guaranteeing the asymptotic stability of \( A \) is that all \( B_i = A_i^T + A_i \) are asymptotically stable.

**Proof**

If \( A_i^T + A_i \) is asymptotically stable then it is negative definite (symmetric matrix). Then \( A_i^T + A_i \) is negative definite. Hence \( A \) is asymptotically stable (Lyapunov equation).

**Theorem 2**

If some \( \lambda_{\min}(B_i) \geq 0 \), then \( A \) is not asymptotically stable.

**Proof**

If some \( \lambda_{\min}(B_i) \geq 0 \), then \( A_i^T + A_i \geq 0 \). Then \( A_i^T + A_i \) is positive semidefinite for \( \beta_j = 0, j \neq i \), which means that \( A \) is not asymptotically stable.

**Theorem 3**

A sufficient condition for the asymptotic stability of the interval matrix \( R \) is that every \( B_i \) is asymptotically stable.

\[ B = \frac{S^T + S}{2} \]

where

\[
\begin{cases} 
R = [r_{ij}], & l_{ij} \leq r_{ij} \leq k_{ij}, \quad i, j = 1, 2, \ldots, n \\
S = [s_{ij}], & s_{ij} = l_{ij} \text{ or } k_{ij}, \quad i, j = 1, 2, \ldots, n
\end{cases}
\]

Received 19 January 1988.

† Department of Automatic Control, Swiss Federal Institute of Technology, 8092 Zürich, Switzerland.
Correspondence

Proof
This follows directly from Theorem 1. Here
\[ R = \sum \beta_i S_i, \quad \beta_i \geq 0 \quad \text{and} \quad \sum \beta_i = 1 \]

2. Extension of the result for Hurwitz stability
If we consider \( B_i = A_i^T P + PA_i \) where \( P \) is positive definite, Theorem 1 and Theorem 3 remain the same with the new \( B_i \). For Theorem 2, it is sufficient that some \( B_i \) are positive.

3. Similar results for Schur stability
Let \( R, S \) be defined as above.
Let \( B = S^T S - I \), then we obtain the following theorem.

Theorem 4
A sufficient condition for the asymptotic stability of the interval matrix \( R \) is that every \( B \) is negative definite.

Proof
If \( B \) is negative definite then \( \|S\|_2 < 1 \). However, according to a theorem by Mori and Kokame (1987), if \( \|S\|_2 < 1 \) for all \( S \), then \( R \) is Schur-stable.

Generalization
If \( B = S^T PS - P \), then the statement in Theorem 4 is also valid (Mori and Kokame 1987). Theorem 4 is a special case of the result of Mori and Kokame (1987).

4. Concluding remarks
It has been shown that the results of Jiang (1987) can be obtained in a very simple way using the Lyapunov equation. It can easily be extended and similar results can be obtained for discrete systems.

References