On the Terminology Relationship Between Continuous and Discrete Systems Criteria

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In this letter definitions of Analog, Counterpart, and Equivalent criteria between discrete and continuous systems are presented. Also, these definitions are illustrated by examples from the various criteria.

INTRODUCTION

In view of the emergence of discrete system theory in recent years, one notices the developments of many new criteria and concepts in this field. Some of these criteria have in many instances followed those developed for continuous systems. Thus certain relationships exist between these criteria for both systems.

In the literature, the terminology of the discrete system criteria as related to the continuous ones is not well defined. Of those used, there exist no unanimous agreement to their meaning. In this letter, we will attempt to define such terminology in terms of analog, counterpart, and equivalent criteria. Several examples, well known in the literature, are illustrated for these definitions.

Though these definitions are not inclusive for all cases, however, we believe that they constitute an initial attempt in clarifying the newly developed discrete criteria.

DEFINITIONS

Analog discrete criteria: In this definition we mean that the criterion in the discrete case has the same form as the continuous one.

Counterpart discrete criteria: In this definition we mean that the criterion in the discrete case has the same strength, i.e., the same information as the continuous one.

Equivalent discrete criteria: In this definition we mean that the criterion has the same form and strength as the continuous one, i.e., analog plus counterpart.

Examples

1) Analog: It is known that for Hurwitz polynomials all the coefficients are positive; for the discrete case, Mansour [1] had obtained such conditions on the coefficients which have the same form as for the Hurwitz polynomial.

Counterpart: In this case, Unbehauen [2] and Anderson and Jury [3] have obtained through the bilinear transformation similar conditions on the coefficients of the discrete polynomials which have the same strength as for the Hurwitz polynomial.

2) Analog: In this example, Tsyplin [4] obtained a nonlinear discrete stability criterion which has the same form as Popov’s criterion [5]. It is of the following form:

$$\text{Re} \left[ \left( 1 + a \left( 1 - e^{-i\omega} \right) G(e^{i\omega}) \right) + \frac{1}{k} \right] > 0, \quad \forall \omega \in [0, 2\pi]. \quad (1)$$

Counterpart: In this example, Jury and Lee [6] have obtained a criterion for the discrete case which has the same strength as Popov’s criterion for the continuous case. It is given as follows:

$$\text{Re} \left[ G(z) \left( 1 + q(z - 1) \right) \right] + \frac{1}{k} - \frac{K_i}{2} \left[ (z - 1) G(z) \right]^2 > 0, \quad \forall |z| = 1 - |e^{i\omega}|, \quad \forall \omega \in [0, 2\pi]. \quad (2)$$

3) Counterpart: In this example, the Jury–Marden [7] table form of the stability has the same strength as Routh’s criterion [8], that is, the same information is obtained for both the respective cases.

Equivalent: In this example, Bistritz’s table form [9], [10] has the same form and strength as the Routh’s criterion. Hence, it is considered as the discrete equivalent of the Routh’s criterion.

Conclusions

In this letter three definitions are introduced which indicate the relationships between discrete and continuous criteria. Namely, analog, counterpart, and equivalent. These are by no means the only ones or the best definitions. However, for initial terminology, it seems reasonable to introduce them.

There exists other discrete criteria which do not fit any of these definitions. For instance, the discrete maximum principle falls in such a criterion. It is hoped that this letter would provoke other remarks in obtaining unanimous definitions.

REFERENCES