MODEL REDUCTION OF DISCRETE-TIME SYSTEMS USING THE SCHWARZ CANONICAL FORM

Indexing terms: Discrete-time systems, Modelling

The Schwarz canonical form description of a linear discrete-time system will be used to derive reduced-order models which are stable if the original system is stable. Further, the steady-state response of the models to a step input is equal to that of the system.

The Schwarz canonical form for discrete systems was first introduced by Mansour to show the relation between the Schur-Cohn stability conditions and the second method of Lyapunov. The system matrix used in this letter is similar to that in Reference 1.

It can be shown that any controllable, single-input/single-output, linear discrete system of order \( n \) can be described as:

\[
x(k+1) = Fx(k) + gu(k) \tag{1a}
\]

\[
y(k) = h^T x(k) \tag{1b}
\]

where

\[
F = \begin{bmatrix}
-\Delta_1 & (1 - \Delta_1^2) & 0 & \ldots & 0 \\
-\Delta_2 & -\Delta_1 \Delta_2 & (1 - \Delta_2^2) & \ldots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
-\Delta_{n-1} & -\Delta_1 \Delta_{n-1} & -\Delta_2 \Delta_{n-1} & \ldots & (1 - \Delta_{n-1}^2) \\
-\Delta_n & -\Delta_1 \Delta_n & -\Delta_2 \Delta_n & \ldots & -\Delta_{n-1} \Delta_n \\
\end{bmatrix}
\]

\[
g = \begin{bmatrix}
1 \\
0 \\
\vdots \\
0 \\
0
\end{bmatrix}
\]

\[
h^T = [h_1, h_2, \ldots, h_n]
\]

\( F \) is the Schwarz canonical form. \( \Delta_1, \Delta_{n-1}, \ldots, \Delta_1 \) are the absolute terms of the characteristic polynomial of degree \( n \) and the reduced polynomial of degree \( n - 1, \ldots, 1 \), respectively, obtained from the Schur–Cohn stability table. The Schwarz matrix was used for stability studies in References 1, 2 and 3. If the system (eqn. 1) is stable, then \( |\Delta_1| < 1, |\Delta_2| < 1, \ldots, |\Delta_n| < 1 \) and it can be shown that \( \Delta_2 \leq 1 \). Hence, the reduced model is stable. It can also be shown that, if the transfer function of the system (eqn. 1) is \( G(z) \) and that of the reduced-order model (eqn. 4) is \( G(z) \), then \( G(1) = G(1) \), i.e. the first time moment of the reduced model is equal to that of the system and so is also the steady-state response to a step input. The reduction can be continued in the same manner described above, i.e. \( \Delta_n \) is deleted, where \( m \) is the model order reached by the last reduction step and \( \Delta_{n-1} \) is modified to \( \Delta_{n-1} = [(\Delta_{n-1} + \Delta_n)/(1 + \Delta_{n-1} \Delta_n)] \). The reduced \( (m - 1) \times (m - 1) \) matrix \( \bar{F} \) takes the Schwarz form with \( \Delta_{n-1} \) replaced by \( \Delta_{n-1} \). \( \bar{G} \) is dimensioned properly and the elements of \( h^T \) are modified to \( h' \) so that

\[
\bar{h}_1 = h_1 - h_0, \bar{h}_2 = h_2 - \Delta_1 h_0, \ldots, \bar{h}_{m-1} = h_{m-1} - \Delta_{m-2} h_0
\]

where

\[
h_0 = \frac{\Delta_m h_m}{1 + \Delta_{m-1} \Delta_m}
\]

The reduction procedure is very simple and so well suited to large-scale systems, and appealing to programming on digital computers.

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References
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