On a Conjecture for the Design of Low-Pass Recursive Filters

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Abstract—Recently, Shenoi and Agrawal [1] discussed the design of low-pass recursive filters using a modified Darlington scheme. A conjecture was made regarding the form of the numerator in the magnitude-squared function. This conjecture was based on two identities satisfied by Chebyshev polynomials. This correspondence provides the proofs for these identities.

I. INTRODUCTION

Shenoi and Agrawal [1, 2] recently proposed a modified Darlington scheme in the design of recursive low-pass filters. In their algorithm, for odd \( N \), the form of the numerator \( \Phi(x) \) in the magnitude-squared function \( S(w) \) was claimed to be of the form \( (1 + x) Q(x) \) where \( Q(x) \) is of degree \( \lfloor N/2 \rfloor \). (Here, \( \lfloor x \rfloor \) denotes the largest integer contained in \( x \).) This form of \( \Phi(x) \), for odd \( N \), depends on two identities involving Chebyshev polynomials of the first kind [2]. These two identities are

\[
T_{2N}(x) + 1 = 2[T_N(x)]^2
\]

and

\[
T_{2N+1}(x) + 1 = (1 + x) \left[ 2 \sum_{i=0}^{N} (-1)^i T_{N-i}(x) + (-1)^{N+1} \right]^2
\]

where \( T_i \) is the \( i \)th degree Chebyshev polynomial of the first kind. The first identity is a special case of a well-known result. The second identity has not, to the author’s knowledge, been proven. An inductive proof is provided here.

II. THE IDENTITIES

Identity 1): \( T_{2N}(x) + 1 = 2[T_N(x)]^2 \)

Since \( T_N(x) = \cos n \theta \) for \( x = \cos \theta \) and \( \cos n \theta \cos m \theta = 1/2 \left[ \cos (m+n) \theta + \cos (m-n) \theta \right] \), one gets

\[ T_n(x) T_m(x) = \frac{1}{2} \left[ T_{m+n}(x) + T_{m-n}(x) \right]. \]

Letting \( m = n = N \) in (3) and noting that \( T_0(x) = 1 \), (1) is obtained.

Identity 2):

\[
T_{2N+1}(x) + 1 = (1 + x) \left[ 2 \sum_{i=0}^{N} (-1)^i T_{N-i}(x) + (-1)^{N+1} \right]^2.
\]

It is readily shown that the identity (2) holds for \( N = 0, 1 \). Let the identity be valid for \( N = k \), i.e.,

\[
T_{2k+1}(x) + 1 = (1 + x) \left[ 2 \sum_{i=0}^{k} (-1)^i T_{k-i}(x) + (-1)^{k+1} \right]^2.
\]

It will be shown that (2) is valid for \( N = k + 1 \), i.e.,

\[
T_{2k+3}(x) + 1 = (1 + x) \left[ 2 \sum_{i=0}^{k+1} (-1)^i T_{k+1-i}(x) + (-1)^{k+2} \right]^2.
\]

Proof:

\[
\text{LHS} = T_{2k+3} + 1 = 2xT_{2(k+1)} - T_{2k+1} + 1
\]

\[
= 2x[2T_{k+1}^2 - 1 - (T_{2k+1} + 1) + 2], \text{ using (2)}
\]

\[
= (1 + x)(4T_{k+1}^2 - 4T_{k+1} - 1) + 2 - 2x.
\]

\[
\text{RHS} = (1 + x) \left\{ 2 \sum_{i=0}^{k+1} (-1)^i T_{k+1-i} + (-1)^{k+2} \right\}^2
\]

\[
= 4(1 + x) T_{k+1}^2 - 4(1 + x) T_{k+1} \left( 2 \sum_{i=0}^{k} (-1)^i T_{k-i} + (-1)^{k+1} \right)^2.
\]

Using (4), one arrives at a new identity to be proved, i.e.,

\[
-4T_{k+1}^2 + 2 - 2x = -4(1 + x) T_{k+1} \left( 2 \sum_{i=0}^{k} (-1)^i T_{k-i} + (-1)^{k+1} \right)^2
\]

\[
+ (-1)^{k+1} \right\} + (1 + x) \left( 2 \sum_{i=0}^{k} (-1)^i T_{k-i} + (-1)^{k+1} \right)^2.
\]

From the three-term recurrence relation,

\[
T_{k+1} = 2xT_k - T_{k-1}
\]

one can deduce

\[
T_{k+1} = 2(x + 1) \sum_{i=0}^{k} (-1)^i T_{k-i} + (-1)^{k+1} (1 + x) - T_k.
\]

Also, if \( m = k + 1, n = k \) in (3), one obtains

\[
4T_{k+1}^2 + 2 - 2x = 4(1 + x) T_{k+1} \left( 2 \sum_{i=0}^{k} (-1)^i T_{k-i} + (-1)^{k+1} \right)^2
\]

Replacing one \( T_{k+1} \) factor on the LHS of (6) by (7) and using (8), the identity (6), and thus the identity (2), is obtained.

REFERENCES


Corrections to “Evaluation of Quantization Error in Two-Dimensional Digital Filters”

P. Agathoklis, E. I. Jury, and M. Mansour

1) Delete footnote 1 on p. 273.
2) In the second line of the second paragraph in the Abstract (p. 273), \( u^2 \) should be \( U^2 \).

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3) On p. 278
\[ \sum_{m=0}^{30} \sum_{n=0}^{30} |y(m, n)| = 3145 \]
should be
\[ \sum_{m=0}^{30} \sum_{n=0}^{30} |y(m, n)| = 3.145 \]
and
\[ \sum_{m=0}^{100} \sum_{n=0}^{100} |y(m, n)| = 3772 \]
should be
\[ \sum_{m=0}^{100} \sum_{n=0}^{100} |y(m, n)| = 3.772. \]

Book Reviews


To many of us, loudspeakers are innately fascinating devices. In assessing their performance, we ritualistically focus our attention and critically listen to the music that emerges. At times, we stare at the loudspeaker system and secretly experience the bewilderment of that famous dog Nipper who, upon hearing his master’s voice, curiously looks down the horn of the gramophone. How convincing is the sonic illusion? How good are the loudspeakers? How do they work? Does such subjective evaluation make each auditor an expert? What are the objective measures of performance? What is the current state of the loudspeaker art?

Mr. Martin Colloms has done a superb job in presenting a well-balanced, objective report on the subject of contemporary high performance loudspeakers. His book consists of eight chapters, each about 30 pages in length. It contains more than 160 interesting figures and photographs, several worthwhile tables and graphs, and a good number of important design formulas. Explanations and derivations are economical and concise. The references and bibliography are adequate, but not exhaustive. In spite of the small type, the design of the book is good. This is not a handbook nor an overthick hobbyist’s magazine, but a sensible, well-written technical compendium of information for the nonspecialist. The book is written at a level understandable to junior or senior engineering students.

In the Preface, the author explains, “A high quality loudspeaker is required to reproduce sound with sufficient fidelity to satisfy a critical audience when fed with an accurate electrical signal.” Chapter 1 presents a general review of major advances in loudspeaker technology during the last decade. Chapter 2 discusses theoretical aspects of diaphragm radiators while Chapter 3 is devoted to the actual performance of practical diaphragms. Chapter 4 considers acoustic loading and low-frequency system analysis including closed and vented box systems as well as transmission-line and horn loading. Chapter 5 deals with moving-coil direct radiator drivers. Chapter 6 investigates both passive and active filters used for crossovers. Chapter 7 is devoted to very practical aspects of loudspeaker enclosures. Chapter 8 addresses itself to the important subject of loudspeaker assessment including specifications, standards, objective measurements, linear and nonlinear distortions, and subjective evaluation.

High Performance Loudspeakers certainly would be a worthwhile addition to a university library or technical educator’s bookshelf. This book will easily satisfy some of the early curiosities of engineering students. Since the human ear is the ultimate receiver in many practical communication systems, those workers interested in reproduction of speech or music also are advised to acquire and read this timely book. Another recent publication of considerable merit and one that this reviewer finds complementary to the book subject is the comprehensive collection of more than 60 technical papers entitled: Loudspeakers: An Anthology of Articles on Loudspeakers from the Pages of the Journal of the Audio Engineering Society, Vol. I-Vol. 25, 1953-1977 (available from: The Audio Engineering Society, Inc., 60 East 42nd St., New York, NY 10017).

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