A Short Note on Another Characterization of Prime Numbers

Mohamed Mansour

ETH Zurich

www.control.ethz.ch/info/people/mansour

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Abstract

In [2] two new related characterizations of prime numbers were introduced. In this note a third characterization is introduced which leads to the same primness criterion as the one obtained from the arithmetic series of composites [1].

keywords: prime numbers

1 Introduction

In [2] two new related characterizations of prime numbers were introduced. In this note a third characterization is introduced which leads to the same primness criterion as the one obtained from the arithmetic series of composites [1].

Sections 2 and 3 shortly discuss the possibility of expressing odd and even composite numbers as sums of neighbouring odd numbers (NONs) and neighbouring even numbers (NEN). Making use of NONs in section 4, a primeness condition is derived, which coincides with the one derived from the arithmetic series of composites [1].

2 Sum of neighbouring odd numbers

The new characterization depends on the sum of NONs. The sum of \( n \) NONs is the square of \( n \) i.e. all the sums of NONs beginning with 1 give all the squares of natural numbers.

\[
1 + 3 + 5 + 7 + 9 + \ldots = 4 + 9 + 16 + 25 + \ldots
\]

Any odd composite number other than the squares is given by the sum of 3, 5, 7, 9, \ldots NONs.

\[
x = n^2 + 2nk \quad k = 0, 1, 2, 3, \ldots \quad (1)
\]

An odd prime number is not a sum of NONs, which is the new characterization of primes.

3 Sum of neighbouring even numbers

Similarly for any even \( n \) NENs the sum \( y \) is even and is given by

\[
y = n^2 + n + 2nk \quad k = 0, 1, 2, 3, \ldots \quad (2)
\]

Equations (1) and (2) with even \( n \) give all the even composites. An even composite is either a sum of NONs or a sum of NENs or both. Only the even number 2 is neither a sum of NONs nor a sum of NENs.

A criterion can be formulated as follows:

Characterization 1. Odd prime numbers cannot be sums of NONs. The even prime 2 is not a sum of either NONs or NENs. All other natural numbers (except 1) are sums of either NONs, NENs or both.
4 Primeness Condition

A sum of NONs can be written as \( a + \cdots + b \). Then

\[
x = \frac{a + b}{2} \times \left[ \frac{b - a}{2} + 1 \right].
\]

Let \( c = \frac{a+b}{2} \) and \( k = \frac{a-1}{2} \), then

\[
x = c^2 - 2kc.
\] (3)

Equation (3) is the same as the equation for composites with \(-c\) instead of \(n\). Its solution for \(c\) is given by (4).

\[
c = k + \sqrt{k^2 + x}
\] (4)

Therefore \(k^2 + x\) should be square for \(x\) to be composite (apart from the trivial solution \(k = \frac{a-1}{2}\)).

Conclusions

A new characterization of primes dependent on sums of NONs and NENs is developed. This is a completion of [2].

References
