FUZZY-TUNED STATE-FEEDBACK CONTROL
OF A NON-HOLONOMIC MOBILE ROBOT

E. BADREDDIN and M. MANSOUR

Automatic Control Lab. - Swiss Federal Institute of Technology
ETH-Zentrum - CH-8092 Zurich - Switzerland

Abstract: A state-feedback controller for the position and orientation of a non-holonomic mobile robot is analysed, designed and implemented. The analysis is based on the polar description of the robot’s kinematics in contrast to the Cartesian description used by other authors. It turns out that the origin is stabilizable using a state-feedback which removes the kinematic singularity from the open-loop description. The controller performance is enhanced by tuning the feedback gains according to few fuzzy rules. These are so formulated to produce an almost straight-line approach towards the goal position and finally adjusting the orientation to the desired value.

Key words: Mobile robots, Non-holonomic constraints, Position control, Fuzzy-control, State-feedback

1. INTRODUCTION

Lately, the control of non-holonomic vehicles has come into the focus of interest of many researchers, e.g., Samson (1990), d’Andrea-Novel et. al (1991), Campion et. al. (1990). A mobile robot employing conventional wheels underlies non-holonomic constraints since, in the absence of skid, its velocity in the direction of the wheel axis is identically zero. Under these constraints, an autonomous mobile robot ought to control both its position and orientation in order to reach a specified goal posture. Traditionally, a kinematics-based controller is used to track a trajectory described by its position and velocity profiles as functions of time. Obviously, there are several disadvantages to this approach. The computation burden can be very heavy especially when a collision-free trajectory is to be synthesised in the configuration space subject to position, velocity, acceleration and even jerk constraints. The problem becomes worse as the environment gets more cluttered. Further, the trajectory will have to be described by a useful number of parameters allowing its on-line generation. Finally, an -optimal- pre-computed trajectory can render useless under drastically changing dynamics. For these, and other reasons, we chose to restrict our goal specification to its co-ordinates in the floor-frame. The motivation for a fuzzy-tuned controller has its back-ground in the control structure (Badreddin 1989, 1990). Since the collision avoidance layer provides safe motion in a reflexive manner (Badreddin et. al. 1991), this requirement can be dropped in the design of the kinematic controller and there will be no need to pre-plan the whole path in the configuration space. The path-planner would only deliver the posture of sub-goals lying on the path. The trajectory between these positions is determined by the controller gains and the other reactive behaviours. These are to so tuned such that a “satisfactory” trajectory is obtained.

2. PROBLEM STATEMENT

Given a posture-error vector $e = [x \ y \ \phi]'$ where 
$\{x, y, \phi\}$ are the goal co-ordinates –or posture– in the robot reference system, find the control-matrix, if it exists, $K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix}$ with $k_i = k(t, e)$ for $i=1,2, 
\text{j=1,2,3}$ such that the control $u = K.e = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ with $u \in U = \{u | |u| \leq 1 \}$ makes $\lim_{t \rightarrow \infty} e(t) = 0$.

This formulation as a state-feedback is superior to open-loop strategies for obvious reasons.

3. KINEMATIC MODEL

Assume, without loss of generality, that the goal is the origin of the inertial floor-frame (Fig.1) The robot kinematics described in its frame of reference -with the dot standing for time derivative- are given by,

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \left( \phi \times \begin{bmatrix} x \\ y \\ u_1 \end{bmatrix} \right) + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

where $\dot{x}$ and $\dot{y}$ are the linear velocities in the direction of the x- and y-axes respectively, and $\phi$ is the angular velocity about the -vertical- z-axis. But since $u_1 = 0$, the goal-kinematics can be written as:

$$p = Ap + Bu$$

with the state vector $p = [x \ y \ \phi]'$, the control

$$u = [u_1 \ u_2]' \ A = \begin{bmatrix} 0 & -\phi & 0 \\ \phi & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & -1 \end{bmatrix}$$

12th IFAC World Congress, Sydney, Australia (1993) Vol. 5 —577
4. CONTROL LAW

4.1 linear state-feedback

Based on a theorem by Brockett(1983), it has been shown by Samson(1990), Campion(1990), and others that the system in its Cartesian representation is not stabilizable using smooth, static state-feedback. The reason is that, in its Cartesian representation, the linearised system about the origin possesses a critical uncontrollable mode. In addition, the non-linear system possesses less control-variables than state-variables and consequently, the matrix $\hat{B}(p)$ in the system description is rectangular. Time varying and dynamic stabilising feedback has been developed by Samson(1990) and Novel(1991). These strategies suffered from some awesome behaviour such as non-monotonically decreasing distance and heavy—though decaying—oscillations. A hybrid strategy with discontinuous control law has been developed by Pomet(1992) which showed better global behaviour in terms of the traversed trajectory.

Consider the system in its Polar representation while applying the static state-feedback,

$$K = \begin{bmatrix} k_a & 0 \\ k_p & k_s \end{bmatrix}$$

with $k_a$, $k_p$, and $k_s \in \mathbb{R}$.

The closed-loop control-system is then,

$$\dot{p} = -k_p \rho \cos \alpha$$

$$\dot{\alpha} = -k_a \alpha - k_s \phi + k_p \sin \alpha$$

$$\dot{\phi} = -k_a \alpha - k_s \phi$$

It is immediately evident that the origin, $\rho = \alpha = \phi = 0$, is an equilibrium point. The linearised system about the origin can be written,

$$\dot{\phi} = \bar{A} \rho,$$

with

$$\bar{A} = \begin{bmatrix} -k_p & 0 & 0 \\ 0 & -(k_a - k_p) & -k_s \\ 0 & 0 & -k_a & -k_s \end{bmatrix}$$

The stability conditions are easily found to be,

$$k_p > 0$$

$$k_s < 0$$

$$k_a + k_s > k_p, i.e., k_a > k_p - k_s$$

The reason for stabilizability of the origin in this representation, in contrast to the Cartesian one, is that the kinematic singularity at $\rho = 0$ is removed by the state-feedback $u_s = k_p \rho$. Now, the system has only a single input, namely, $u_s$. The linearised single-input system about $\rho = \alpha = \phi = 0$ can be written as,

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\phi} \end{bmatrix} = \hat{B}(p) \begin{bmatrix} u_s \end{bmatrix}$$

It is immediately seen that the unstable mode is controllable and is, therefore, stabilizable.
Thus, we have shown the local asymptotic stability using a non-smooth state-feedback. The proof of
global stability requires the formulation of a suitable
Lyapunov-function. Simulation results for the
manoeuvre of lateral-displacement of the robot under
the proposed linear state-feedback, are shown in
figures 2 and 3. The initial conditions and feedback
gains are set to:

\[ \rho(0) = 1 m, \alpha(0) = \frac{\pi}{2} \text{rad}, \phi(0) = 0 \text{rad} \]

\[ k_{\rho} = 1.0, k_{\alpha} = 5.0, k_{\phi} = -1.5 \]

Fig.2 Lateral displacement manoeuvre.

Fig.3 Translation and rotational command.

4.2 Fuzzy-tuned Controller

A human operator would probably suggest the
following intuitive control strategy:
• First, turn the robot to the goal heading,
• then, drive straight-ahead until you reach the goal
position,
• and finally, turn the robot on-place until the
desired orientation is reached.

This strategy, implicitly, makes use of the kinematic
singularity at the origin which allows changing the
orientation, \( \phi \), without changing the magnitude, \( \rho \), or
angle, \( \alpha \), of the position-error. This strategy can be
translated into state-feedback rules as follows:

**Rule-1:** When \( \rho \) is large, make \( \alpha \) small. Notice that
it is necessary to have \( \alpha = 0 \) to be able to reach the
goal position at all. At \( \rho = 0 \), \( \alpha \) is undefined and is
also no longer interesting.

**Rule-2:** As long as \( \alpha \) is large, move only slowly
towards the goal. This enhances a straight-in
approach to the goal.

**Rule-3:** As \( \rho \) gets smaller, make \( \phi \) small.

Similar "crisp" rules are proposed by Bloch(1990) as
an open-loop control strategy for the "knife-edge"
problem which has the same non-holonomic
constraint as our mobile robot. This control has been
shown to transfer any initial state to the origin.

**Membership Function:** To implement the previous
crisp-rules membership functions are to be chosen for
the terms "small" and "large". The following choice
was made based on the value range of the states:

- small \( \rho : \mu(\rho) = e^{\left(\frac{\rho}{2}\right)} \)
- small \( \alpha : \mu(\alpha) = \cos^2 \alpha \)

**Tuning scheme:** Next, the feedback gains are tuned
according to the previous rules while maintaining the
stability conditions. Such a tuning scheme is, e.g.

\[ K_{\rho} = K_{\rho} \cos^2 \alpha, \quad K_{\alpha} = K_{\alpha}(1 + \sin^2 \alpha) \left(2 - e^{\frac{\rho}{2}}\right) \]

\[ K_{\phi} = K_{\phi}(1 + \sin^2 \alpha) e^{\frac{\rho}{2}} \]

The linearised closed-loop about the origin is the
same as without tuning. The stability conditions hold,
therefore, invariably for the static gains.

The tuning action is summarised in the following table:

<table>
<thead>
<tr>
<th>Magnitude of ( \rho )</th>
<th>Large ( \equiv \infty )</th>
<th>Small ( \equiv 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha ) ↓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Large ( \equiv \pi/2 )</td>
<td>( K_{\rho} = 0 )</td>
<td>( K_{\rho} = 0 )</td>
</tr>
<tr>
<td>( K_{\alpha} = 4K_{\alpha} )</td>
<td>( K_{\alpha} = 2K_{\alpha} )</td>
<td>( K_{\alpha} = 2K_{\alpha} )</td>
</tr>
<tr>
<td>( K_{\phi} = 0 )</td>
<td>( K_{\phi} = 0 )</td>
<td>( K_{\phi} = 0 )</td>
</tr>
</tbody>
</table>

Small \( \equiv 0 \)

\( K_{\rho} = K_{\rho} \)

\( K_{\alpha} = K_{\alpha} \)

\( K_{\phi} = K_{\phi} \)

Fig. 4 Trajectory with the tuned state-feedback
Control Behaviour: The effect is that when \( \rho \) is large \( \alpha \) is practically the only controlled variable. No action is taken to control \( \rho \) or \( \phi \) until \( \alpha \) has become small enough. The robot is now heading towards the goal. Then, both \( \alpha \) and \( \rho \) are controlled simultaneously. The robot moves straight towards the goal. Eventually, \( \rho \) is also reduced to a small value and the controller operates on all three variables with the -almost un-tuned- static gains bringing \( \phi \) as well to zero.

Simulation using this tuning scheme is shown in Fig.4 and Fig.5 for the same manoeuvre, with the same initial gains as for the un-tuned controller and \( \sigma = 1.0 \). The straight-in approach can be clearly seen followed by the typical S-shaped trajectory in the vicinity of the goal.

![Graph](image)

Fig.5 Command of the tuned state-feedback controller

5. CONCLUSIONS AND FINAL REMARKS

In an earlier work by Badreddin(1992), a fuzzy-tuned PI-controller for the posture of a non-holonomic mobile robot has been developed and employed without any explicit stability analysis. Simulations and experiments showed that it worked well for all practical purposes. However, the equilibrium of the \( \alpha \) was \( \pm \pi/2 \) instead of zero which means that the polar position-error can hardly be reduced to zero if this had not been the case before the regulation of the orientation angle \( \phi \) has begun. In other words, the value of the stationary position-error depends heavily on how close to zero \( \alpha \) was. This motivated the search for another tuning scheme without this drawback.

In this paper, we attempt to point-out the following:

1) Despite -or because of- the kinematic singularity at the origin, the polar representation of non-holonomic mobile robot kinematics seems to be better suited for control and stability analysis than the Cartesian one. The kinematic singularity is removed by feedback of the polar position-error.

2) Although the proposed feedback is discontinuous at \( \alpha = \pm \pi/2 \) no practical problems were encountered thanks to the word-length and exception-handling capabilities of modern computers.

3) An intuitive set-up of fuzzy-rules can easily be combined with the “standard” solution to enhance the performance. Model knowledge are mandatory in this case. This emphasises the role of fuzzy-set theory as a “higher-level” formalism which, when laid over a standard “low-level” solution, can significantly improve the results obtained by one level alone.

4) The fuzzy-tuned controller proposed here can be regarded as the fuzzification of a variable structure controller which controls only two of the three states at a time and hence making \( \hat{B}(\rho) \) rectangular.

5) Global stability remains an open issue. However, during numerous simulations and practical experiments no instability were noticed.

Finally, it does not go without saying that other tuning rules and different implementations can be found to this problem. Also, practical experiments were necessary to adjust the gain values and the sharpness of the membership functions since the dynamics were not considered neither in the stability analysis nor in the simulation.

The controller is implemented in our mobile robot, RAMSIS, and reliably serving since three years.

6. REFERENCES


